Magnetohydrodynamic Convective Flow of a Walters'B Fluid through a Non–Homogeneous Porous Medium with Soret Effect.

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Abstract
Radiation absorption and chemical reaction effects on unsteady Magnetohydrodynamic free convective flow of a visco-elastic fluid past a porous medium in the presence of Soret effect is considered. A uniform magnetic field is assumed to be applied transversely in the direction of the flow. The set of non-linear partial differential equations is transformed into ordinary differential equations by super imposing a solution with steady and unsteady part. The set of ordinary differential equations is solved using the perturbation method. The expressions for velocity, temperature, species concentration, skin friction, Nusselt number and Sherwood number are obtained. The effects of numerous physical parameters on the above flow quantities are studied with the help of graphs. The result show that the velocity profile increases with an increase in radiation, Schmidt number, elastic parameter, permeability, Soret, while chemical reaction decreases the velocity profile. Temperature profile increases for increase in thermal radiation, heat source, while increase in Prandtl number and frequency of oscillation reduces the temperature of the system. Increase in chemical reaction reduces the concentration of species in the boundary layer.

Keywords: *Magnetohydrodynamics, thermal radiation, Soret, visco-elastic fluid, porous medium.*

1. Introduction:

Magnetohydrodynamic (MHD) free convection fluid flow frequently occur naturally. Here, fluid passes through porous medium and are applied in the field of science and technology for example in studying ground water resources and movement of natural gas and water through the oil reservoirs. Visco-elastic fluid flows are encountered in numerous areas of petrochemical, biomedical and environmental engineering, including polypropylene coalescence sintering, dynamically-loaded bearings and geological flows. Ezzat and Abd-Elaal (2011) studied combined heat and mass transfer for unsteady MHD flow of perfect conducting micro polar fluid with thermal radiation. Waqar et al (2015) investigated MHD stagnation point flow and heat transfer impinging on stretching sheet with chemical reaction and transpiration. Visco-elastic flows and transport phenomena arise in numerous areas of chemical, industrial process, bio - systems, food processing and biomedical engineering. These include the rheology of liquid crystal precursors employed in the manufacture of carbon super fibers, and oil emulsion processing, paper coating rheo-reactor processing, propulsive ciliary transport of respiratory airway mucus, thermo-capillary bubble dynamics in weakly elastic fluids, rheo-reactor phosphatation flows, flour rheology, magonnaise elastic-viscous flows, Xanthan gum hydrogel flows, and polygalacturonase-based food stuff. Ezzat and Abd-Elaal (1997) studied the state space approach to visco-elastic fluid flow in a hydromagnetic fluctuating boundary layer.

Makanda et al (2013) studied the natural convection of visco-elastic fluid from a cone embedded in a porous medium with viscous dissipation using successive linearization method (SLM). Magdy and Abd-Elaal (1997) discussed free convection effects on a visco-elastic boundary layer flow with one relaxation time through a porous medium. The discussion of the effect of cooling and heating on a visco-elastic conducting fluid was illustrated. Convection in porous medium has applications in geo-thermal energy recovery, oil extraction, thermal energy storage and flow through filtering devices. From technological point of view, MHD free convection flows have significant applications in the field of Stellar and planetary magnetosphere, aeronautics, chemical engineering and electronics on accord of their varied importance.

A Newtonian fluid flow over a linear stretching surface was first time considered by Crane (1970). Wang and Anuar (1988) studied the steady flow of a viscous and incompressible fluid outside a stretching cylinder in an ambient fluid at rest. Prasad and Subhas (2003) have investigated on the diffusion of a chemically reactive species of a non-Newtonian fluid immersed over a stretching sheet. MHD free convective fluid flows frequently occur in natural world. Fluid passing through porous medium are of great interest nowadays. Chaudhary et al (2006) considered Hall Effect on MHD mixed convective flow of visco-elastic fluid past an infinite vertical porous plate with mass transfer and radiation. Kesavaiah et al (2011) studied and presented effects of chemical reaction and radiation absorption on unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate embedded in porous medium with heat source and suction. Reddy et al (2013) considered effects of chemical reaction and radiation absorption on unsteady magneto hydrodynamic double diffusive convective flow of viscous fluid past a semi-infinite porous plate. Raju et al (2011) investigated radiation and mass transfer effects on a free convective vertical surface. Muthukumaraswamy et al (2001) analyzed first order chemical reaction on flow past an impulsively started vertical plate with uniform heat and mass flux. Reedy et al (2012) considered chemical reaction and radiation effect of unsteady MHD free convection flow near a moving vertical plate. Raju et al (2010) studied radiation effect on MHD flow through a porous medium with variable temperature or variable mass diffusion. Recently, Ravikumar and Raju (2014) investigated combined effect of heat absorption and MHD on convective Rivlin-Ericksen flow past a semi-infinite vertical porous plate. Venkateswarlu and Satya (2014) had presented chemical reaction and radiation absorption effects on the flow and heat transfer of a nano fluid in a rotating system. In view of increasing importance of visco-elastic fluids in engineering practice, such a problem involving Walters-B fluid flow is investigated here. Here, we examine quantitatively the effect of viscosity of the fluid on the flow and heat transfer on the walls of the channel for the problem under discussion.

2. Mathematical Formulation;

We consider an unsteady hydromagnetic free convection, incompressible viscous flow of an electrically, radiation absorbing chemically reacting visco-elastic fluid through a semi-infinite plate. We chose a co- ordinate system with x'-axis along the plate and y'-axis perpendicular to the plate. The infinite length of the plate makes the flux variable function of y and t only. A uniform magnetic field is applied perpendicular to the plate with the magnetic Reynolds number assumed small so that the induced magnetic field and electric field are negligible. The flow is driven by buoyancy force arising from temperature difference between the wall and the medium. We consider the permeability of the porous medium in the form

$$k'(t) = k'_p(1 + \varepsilon e^{i\omega' t'} \quad (1)$$

where $k'(t), k_p^{'}$ is porosity, ω' is frequency of oscillation t' is time, ε is a small positive constant, $\varepsilon << 1$.
Under the assumptions and the Boussinesq approximation, the equation for continuity, momentum, energy and concentration are:

$$\frac{\partial v'}{\partial y'} = 0 \tag{2}$$

$$\frac{\partial u'}{\partial t'} + v\frac{\partial u'}{\partial y'} = v\frac{\partial^2 u'}{\partial y^2} + g\beta_T(T' - T'_\infty) + g\beta_c(C' - C'_\infty) - \frac{\sigma\beta_0^2}{\rho}u' - \frac{\upsilon u'}{k'_p(1+\varepsilon e^{i\omega' t'})} - \frac{k}{\rho}\left\{\frac{\partial^3 u'}{\partial t\partial y'^2} + \frac{\upsilon\partial^3 u'}{\partial y'^3}\right\} \tag{3}$$

$$\frac{\partial T'}{\partial t'} + v\frac{\partial T'}{\partial y'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + S'(T' - T'_\infty) - \frac{\partial q_r^{'}}{\partial y'} \tag{4}$$

$$\frac{\partial C'}{\partial t'} + v\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} - k^*(C^l - C^l_\infty) + \frac{D_m k_T}{T_m}\frac{\partial^2 T'}{\partial y'^2} \tag{5}$$

The boundary conditions for velocity, temperature and concentration fields are:

$$u = o;\quad T' = T'_\infty + \varepsilon(T_w^{'} - T_\infty^{'})e^{i\omega' t'};\quad C' = C_\infty^{'} + \varepsilon(C_w^{'} - C_\infty^{'})e^{i\omega' t'};\quad \text{at}\quad y = 0 \tag{6}$$

$$u \to 0;\quad T' \to T_\infty^{'};\quad C' \to C_\infty^{'} \qquad \text{as}\quad y \to \infty \tag{7}$$

The suction velocity at the plate is a function of time defined as;

$$v(t^l) = -v_0(1 + \varepsilon e^{i\omega' t'}) \tag{8}$$

$v_0 > 0$.
We assume that the thermal radiation is present in the form of a non-directional flux and following the case of optically thin gray gas, the radiative flux is defined as

$$q_r = \frac{-4}{3}\frac{\sigma^*}{k*}\frac{\partial T'^4}{\partial y'} \tag{9}$$

where σ^* is the Stefan-Boltzmann constant and k^* is the mean assumption coefficient.
We assume that temperature difference in the flux is small so that equation (9) can be linearized by expanding T'^4 using Taylor series about $T_\infty^{'}$ and after neglecting higher order terms, we have Amos and Israel-Cookey (2015)

$$T'^4 \cong 4T_\infty^{'3}T' - 3T_\infty^{'4} \tag{10}$$

Consequently, using equations (9) and (10), equation (4) becomes

$$\frac{\partial T'}{\partial t'} + v\frac{\partial T'}{\partial y'} = \frac{k\partial^2 T'}{\rho C_p \partial y'^2} + S^*(T_w^l - T_\infty^l) + \frac{16\sigma^* T_\infty^{l3}}{3k^*}\frac{\partial^2 T^1}{\partial y^{12}} \tag{11}$$

To present the governing equations and their boundary conditions in dimensionless forms, the following non-dimensional parameters are needed.

$$t = \frac{v_0^2 t'}{4\upsilon};\quad u = \frac{u'}{v_0};\quad y = \frac{v_0 y'}{\upsilon};\quad C = \frac{C' - C_\infty^{'}}{C_w^{'} - C_\infty^{'}};\quad T = \frac{T' - T_\infty^{'}}{T_w^{'} - T_\infty^{'}};\quad S = \frac{\upsilon S'}{v_0},\quad \omega = \frac{4\upsilon\omega'}{v_0^2}$$

$$M = \frac{\sigma\beta_0^2\upsilon}{\rho v_0^2};\quad k_p = \frac{v_0^2 k_p^{'}}{\upsilon^2};\quad p_v = \frac{\upsilon\rho C_p}{k};\quad G_c = \frac{\upsilon g\beta_c(C_w^{'} - C_\infty^{'})}{v_0^3};\quad G_r = \frac{\upsilon g\beta_T(T_w^{'} - T_\infty^{'})}{v_0^3} \tag{12}$$

$$S_c = \frac{\upsilon}{D};\quad R_c = \frac{k_0 v_0^2}{\rho\upsilon^2};\quad k_r = \frac{k'\upsilon}{v_0^2};\quad w = \frac{4\upsilon w'}{v_0^2};\quad Ra = \frac{16\sigma^* T_\infty^{l3}}{3k^* k};$$

In view of (12), the governing equations are

$$\frac{1}{4}\frac{\partial u}{\partial t}-\left(1+\varepsilon e^{i\omega t}\right)\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2}+Gr\,T+Gc\,C-M\,u-\frac{1}{k_p}\left(1+\varepsilon e^{iwt}\right)-$$
$$Rc\left(\frac{1}{4}\frac{\partial^3 u}{\partial t\partial y^2}-\left(1+\varepsilon e^{i\omega t}\right)\frac{\partial^3 u}{\partial y^3}\right), \quad (13)$$

$$\frac{1}{4}\frac{\partial T}{\partial t}-\left(1+\varepsilon e^{i\omega t}\right)\frac{\partial T}{\partial y}=\frac{1}{\Pr}\{1+Ra\}\frac{\partial^2 T}{\partial y^2}+ST \quad (14)$$

and

$$\frac{1}{4}\frac{\partial C}{\partial t}-\left(1+\varepsilon e^{i\omega t}\right)\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial^2 T}{\partial y^2}-k_r C+Sr\frac{\partial^2 T}{\partial y^2} \quad (15)$$

subject to

$$u=0,\quad T=1+\varepsilon e^{i\omega t},\quad C=1+\varepsilon e^{i\omega t} \quad \text{at} \quad y=0 \quad (16)$$
$$u=0,\quad T\to 0,\quad C\to 0 \quad \text{as} \quad y\to\infty \quad (17)$$

The parameters entering the problem are R_c elastic parameter, M, magnetic parameter, Gr, thermal Grashof number, Gc, solutal Grashof number, Sc, Schmidt number, Ra, radiation parameter, K_r, chemical reaction parameter, Sr, Soret and S, heat source.

3. Method of Solution:

Equations (13) – (15) are coupled and solution not easily tractable, hence we adopt the following perturbation expression in their solution.

$$u(y,t)=u_0(y)+\varepsilon e^{i\omega t}u_1(y)+0(\varepsilon^2) \quad (18)$$
$$T(y,t)=T_0(y)+\varepsilon e^{i\omega t}T_1(y)+0(\varepsilon^2) \quad (19)$$
$$C(y,t)=C_0(y)+\varepsilon e^{i\omega t}C_1(y)+0(\varepsilon^2) \quad (20)$$

Application of equations (18) - (20) on equations (13) – (15) yields the following order of approximations.

$$\left(\frac{1+Ra}{Pr}\right)T_0''+T_0'+ST_0=0 \quad (21)$$
$$\left(\frac{1+Ra}{Pr}\right)T_1''+T_1'+\left(S-\frac{i\omega}{4}\right)T_1=T_0' \quad (22)$$
$$\frac{1}{Sc}C_0+C_0'-Kr\,C_0=Sr\,T_0'' \quad (23)$$
$$\frac{1}{Sc}C_1''+C_1'-\left(Kr+\frac{i\omega}{4}\right)C_1=C_0'-Sr\,T_1'' \quad (24)$$
$$Rc\,u_0'''+u_0''+u_0'-\left(M+\frac{1}{k_p}\right)u_0=-Gc\,C_0-Gr\,T_0 \quad (25)$$
$$Rc\,u_1'''+\left(1+\frac{Rc}{4}i\omega\right)u_1''+u_1'-\left(\frac{1}{4}i\omega+M+\frac{1}{k_p}\right)u_1=-\frac{1}{k_p}u_0-Rcu_0'''-u_1'-Gr\,T_1-Gc\,C_1 \quad (26)$$

The boundary conditions are:

$$u_0=0,\quad u_1=0,\quad T_0=1,\quad T_1=0\quad C_0=1,\quad C_1=0 \quad \text{at} \quad y=0 \quad (27)$$
$$u_0=1,\quad u_1=1,\quad T_0=0,\quad T_1=0\quad C_0=1,\quad C_1=0 \quad \text{as} \quad y\to\infty \quad (28)$$

The parameter $Rc << 1$, hence we perform a perturbation of u_0 and u_1, using

$$u_0(y) = u_{00}(y) + R_c u_{01}(y) + 0(R_c^{\,2}) \quad (29)$$

$$u_1(y) = u_{10}(y) + R_c u_{11}(y) + 0(R_c^{\,2}) \quad (30)$$

This yields

$$U_{00}'' + U_{00}' - \left(M + \frac{1}{Kp}\right)U_{00} = -GrT_0 - GcC_0 \quad (31)$$

$$U_{10}'' + U_{10}' - \left(M^2 + \frac{1}{Kp} + \frac{i\omega}{4}\right)U_{10} = U_{00}' - GrT_1 - GcC_1 - \frac{U_{00}}{Kp} \quad (32)$$

$$U_{01}'' + U_{01}' - \left(M + \frac{1}{Kp}\right)U_{01} = -U_{00}''' \quad (33)$$

$$U_{11}'' + U_{11}' - \left(M^2 + \frac{1}{Kp} + \frac{i\omega}{4}\right)U_{11} = -U_{01}' - \frac{1}{Kp}U_{01} - U_{00}''' + \frac{i\omega}{4}U_{10}'' - U_{10}''' \quad (34)$$

The boundary conditions are:

$$u_{00} = 0, \quad u_{01} = 0, \quad u_{10} = 0, \quad u_{11} = 0 \quad \text{at} \quad y = 0 \quad (35)$$

$$u_{00} = 0, \quad u_{01} = 0, \quad u_{10} = 0, \quad u_{11} = 0 \quad \text{as} \quad y \to \infty \quad (36)$$

Solutions for equations (21) – (26) and equations (31) and (34) after some algebra are presented as follows:

$$\begin{aligned} \mathrm{U(y,t)} = {} & (1 - D_6 - D_7)e^{-\beta_5 y} + D_6 e^{-\beta_1 y} + D_7 e^{-\beta_3 y} \\ & + \left\{(1 - D_8 - D_9 - D_{10})e^{-\beta_6 y} + D_8 e^{-\beta_5 y} + D_9 e^{-\beta_1 y} + D_{10} e^{-\beta_3 y}\right\} \\ & + \varepsilon e^{i\omega t}\{(1 - D_{11} - D_{12} - D_{13} - D_{14} - D_{15} - D_{16})e^{-\beta_7 y} + D_{11} e^{-\beta_1 y} \\ & + D_{12} e^{-\beta_2 y} + D_{13} e^{-\beta_3 y} + D_{14} e^{-\beta_4 y} + D_{15} e^{-\beta_5 y} + D_{16} e^{-\beta_8 y} \\ & + R_c\{(1 - D_{17} - D_{18} - D_{19} - D_{20} - D_{21} - D_{22} - D_{23} - D_{24})e^{-\beta_8 y} \\ & + D_{17} e^{-\beta_1 y} + D_{18} e^{-\beta_2 y} + D_{19} e^{-\beta_3 y} + D_{20} e^{-\beta_4 y} + D_{21} e^{-\beta_5 y} + D_{22} e^{-\beta_6 y} \\ & + D_{23} e^{-\beta_7 y} + D_{24} e^{-\beta_8 y}\}\} \end{aligned} \quad (37)$$

$$T(y,t) = e^{-\beta_1 y} + \varepsilon e^{i\omega t}\left\{(1 - D_0)e^{-\beta_2 y} + D_0 e^{-\beta_1 y}\right\} + 0(\varepsilon^2) \quad (38)$$

$$\begin{aligned} C(y,t) = {} & (1 - D_1)e^{-\beta_3 y} + D_1 e^{-\beta_1 y} \\ & + \varepsilon e^{i\omega t}\left\{(1 - D_3 - D_4 - D_5)e^{-\beta_4 y} + D_3 e^{-\beta_3 y} + D_4 e^{-\beta_2 y} + D_5 e^{-\beta_1 y}\right\} \\ & + 0(\varepsilon^2) \end{aligned} \quad (39)$$

Heat Transfer Rate

$$Nu = \left.\frac{\partial T}{\partial y}\right|_{y=0} = -\left[\beta_1 + \varepsilon e^{iwt}\{\beta_2(1 - D_0) + \beta_1 D_0\}\right] \quad (40)$$

Mass Transfer Rate

$$Sh = \left.\frac{\partial C}{\partial y}\right|_{y=0} = -[\beta_3(1 - D_1) + \beta_1 D_1 + \varepsilon e^{iwt}\{\beta_4(1\text{-}D_3\text{-}D_4\text{-}D_5) + \beta_3 D_3 + \beta_2 D_4 + \beta_1 D_5\}] \quad (41)$$

The Skin Friction

$$Cf = -\frac{\partial u}{\partial y}\bigg|_{y=0} = \{\beta_5(1 - D_6 - D_7) + \beta_1 D_6 + \beta_3 D_7 + R_c[\beta_6(1 - D_8 - D_9 - D_{10}) + \beta_5 D_8 + \beta_1 D_9 + \beta_3 D_{10}]\} + \varepsilon e^{iwt}\{\beta_1(1\text{-}D_{11}\text{-}D_{12}\text{-}D_{13}\text{-}D_{14}\text{-}D_{15}\text{-}D_{16}) + \beta_2 D_{12} + \beta_3 D_{13} + \beta_4 D_{14} + \beta_5 D_{15} + \beta_8 D_{16} - R_c[\beta_8(1\text{-}D_{17}\text{-}D_{18}\text{-}D_{19}\text{-}D_{20}\text{-}D_{21}\text{-}D_{22}\text{-}D_{23}\text{-}D_{24}) + \beta_1 D_{17} + \beta_2 D_{18} + \beta_3 D_{19} + \beta_4 D_{20} + \beta_5 D_{21} + \beta_6 D_{22} + \beta_7 D_{23} + \beta_8 D_{24}] \quad (42)$$

4. Analysis of results

The graphical results from the solutions obtained for velocity, temperature, concentration profiles and the rates of heat and mass transfers for various parameter values are presented and discussed.

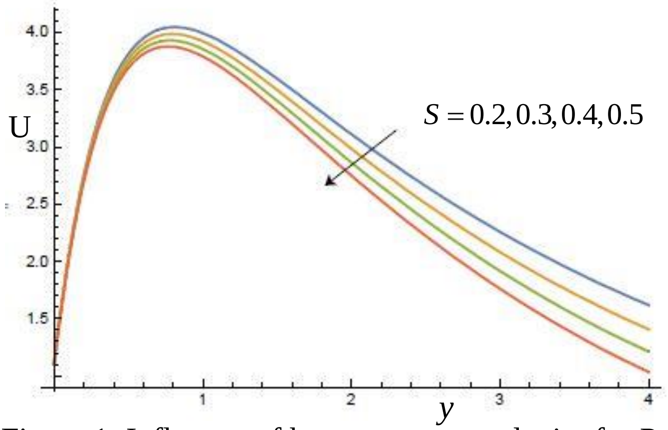

Figure 1: Influence of heat source on velocity for Pr = 0.71, R_a = 0.5, k_r = 0.1, Sc = 0.22, R_c = 0.1, Sr = 0.5, k_p=1.5, ω = 0.1, M = 2, ε = 0.1

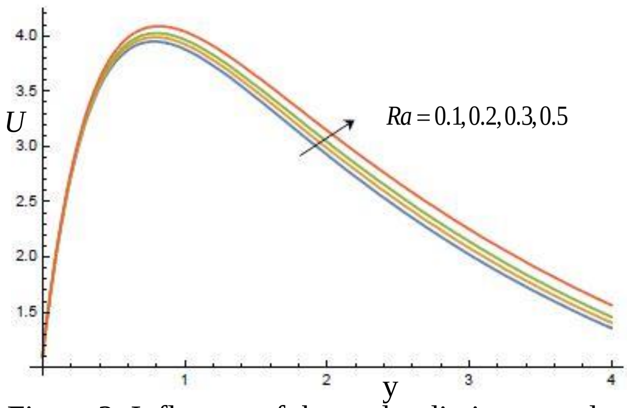

Figure 2: Influence of thermal radiation on velocity for Pr = 0.71, S = 0.01, k_r = 0.1, Sc = 0.22, R_c = 0.1, Sr = 0.5,k_p= 1.5, ω = 0.1, M = 2, ε = 0.1

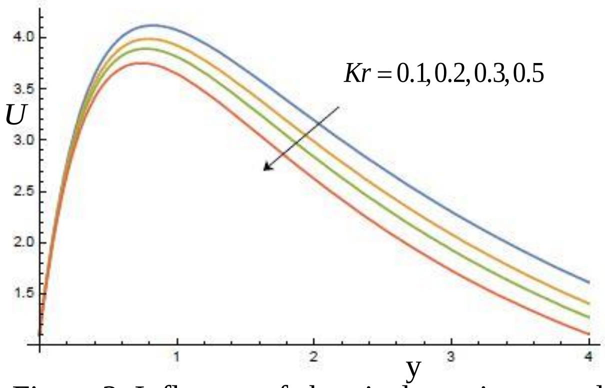

Figure 3: Influence of chemical reaction on velocity for Pr = 0.71, R_a = 0.5, S = 0.01, Sc = 0.22, R_c = 0.1, Sr = 0.5, k_p= 1.5, ω = 0.1, M = 2, ε = 0.1

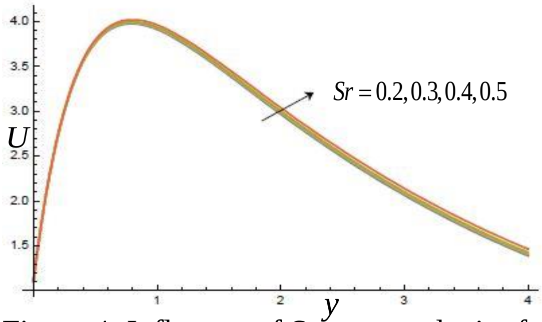

Figure 4: Influence of Soret on velocity for Pr = 0.71, R_a = 0.5, k_r = 0.1, Sc = 0.22, R_c = 0.1, S = 0.01, k_p= 1.5, ω = 0.1, M = 2, ε = 0.1

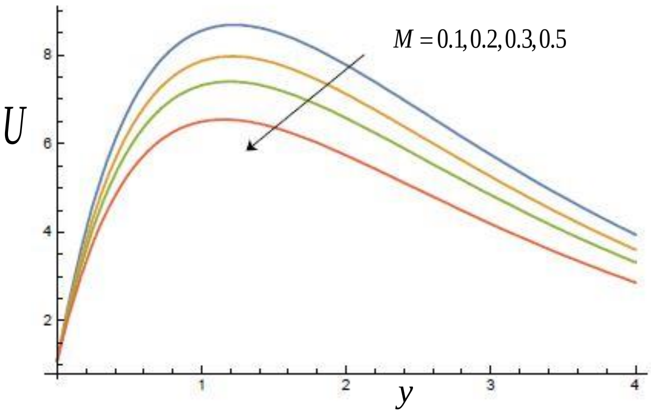

Figure 5: Influence of Magnetic field on velocity for Pr = 0.71, R_a = 0.5, k_r = 0.1, Sc = 0.22, R_c = 0.1, Sr = 0.5, k_p= 1.5, ω = 0.1, S = 0.01, ε = 0.1

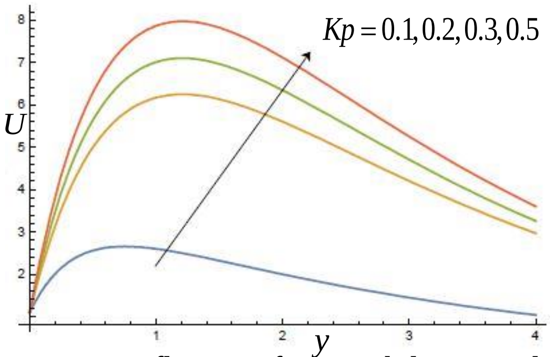

Figure 6: Influence of permeability on velocity for Pr = 0.71, R_a = 0.5, k_r = 0.1, Sc = 0.22, R_c = 0.1, Sr = 0.5, M= 2, ω = 0.1, S = 0.01, ε = 0.1

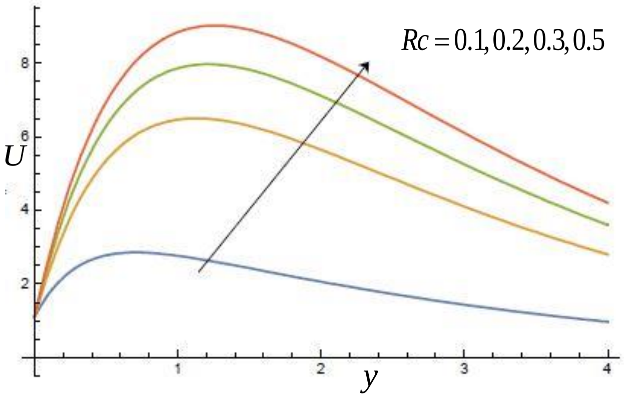

Figure 7: Influence of elastic parameter on velocity for Pr = 0.71, R_a = 0.5, k_r = 0.1, Sc = 0.22, M =2, Sr = 0.5, k_p= 1.5, ω = 0.1, S = 0.01, ε = 0.1

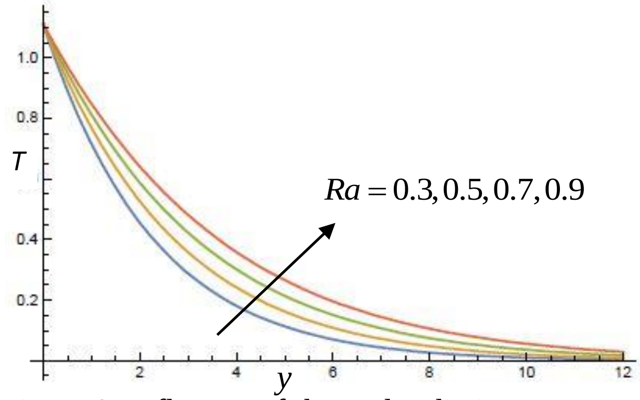

Figure 8: Influence of thermal radiation on Temperature for Pr = 0.71, ω = 0.2, ε = 0.1, n = 0.1

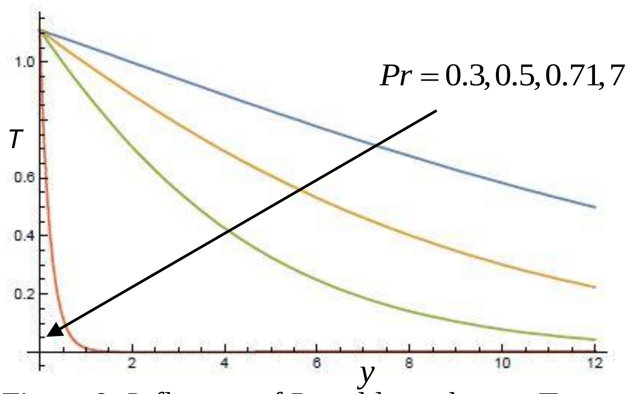

Figure 9: Influence of Prandtl number on Temperature for Ra = 0.5, ω = 0.2, ε = 0.1, n = 0.1, S = 0.01

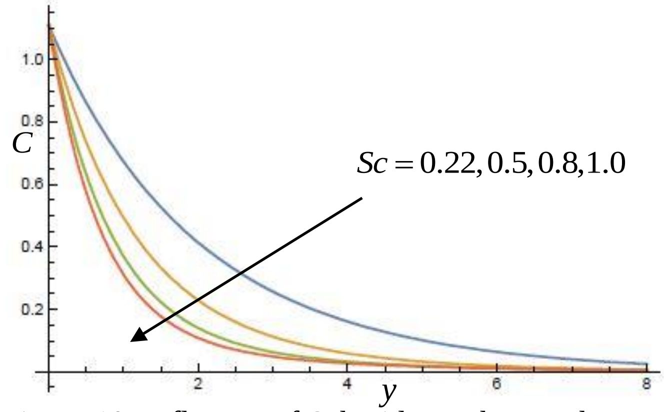

Figure 10: Influence of Schmidt number on the concentration for k_r = 0.1, M = 2, ε = 0.1, n = 0.1, Sr = 0.01

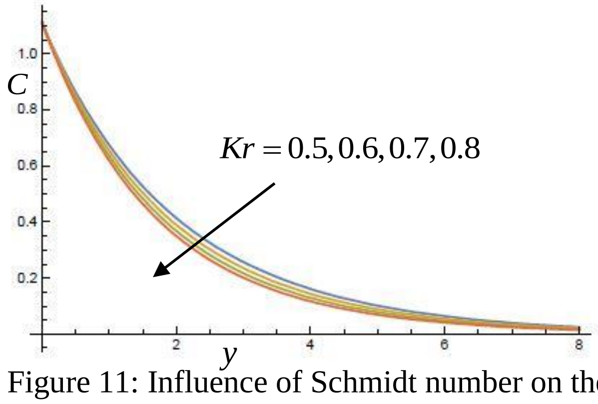

Figure 11: Influence of Schmidt number on the concentration for Pr= 0.71, ω = 0.1, ε = 0.1, n = 0.1,

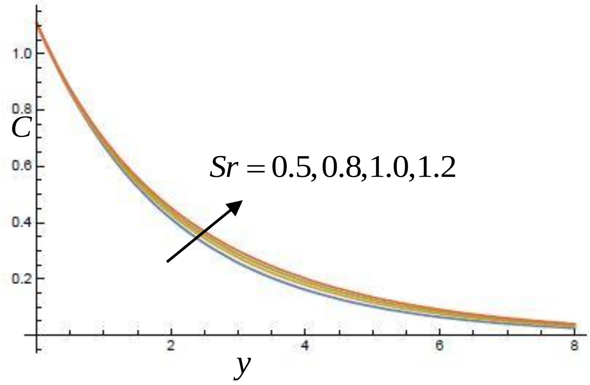

Figure 12: Influence of Soret on the concentration for k_r = 0.1,Pr = 0.71, ε = 0.1, n = 0.1, M =2

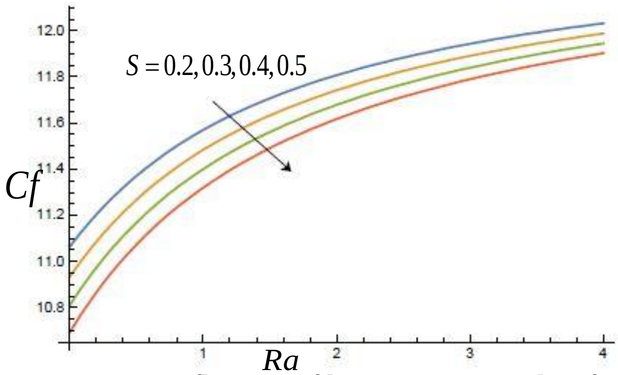

Figure 13: Influence of heat source on Skin friction for Pr = 0.71, k_r = 0.1, Sc = 0.22, R_c = 0.1, Sr = 0.5, k_p= 1.5, Gr = 10, Gc = 10, M = 2, ω = 0.1, ε = 0.1

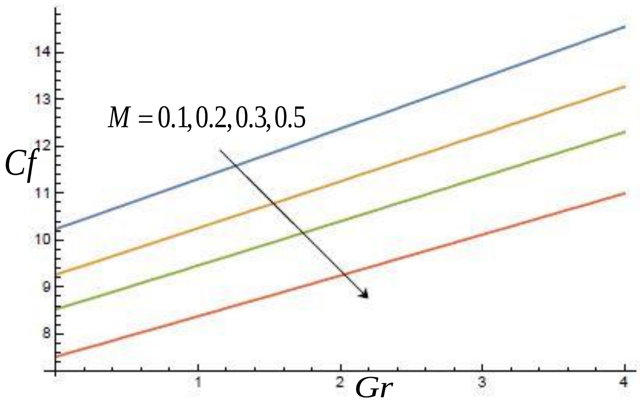

Figure 14: Influence of Magnetic field on Skin friction Pr = 0.71, k_r = 0.1, Sc = 0.22, R_c = 0.1, Sr = 0.5, k_p= 1.5, Gr = 10, Gc = 10, S =0.01, ω = 0.1, ε = 0.1

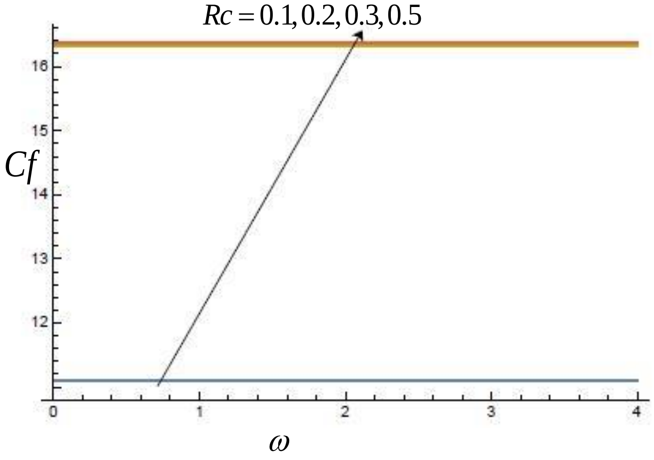

Figure 15: Influence of elastic parameter on Skin friction Pr = 0.71, k_r = 0.1, Sc = 0.22, R_c = 0.1, Sr = 0.5, k_p= 1.5, Gr = 10, Gc = 10, M = 2, S = 0.01, ε = 0.1

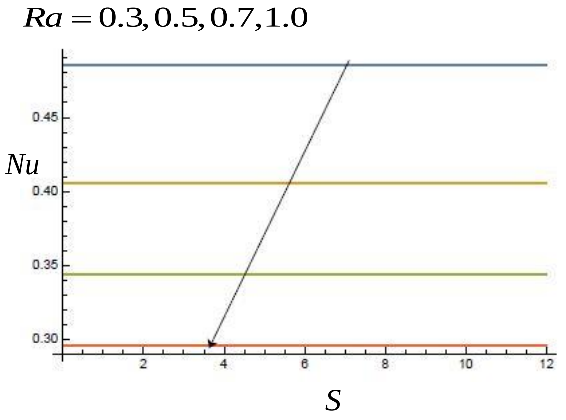

Figure 16: Influence of thermal radiation on heat transfer for Pr = 0.71, ω = 0.2, ε = 0.1, n = 0.1, S = 0.01

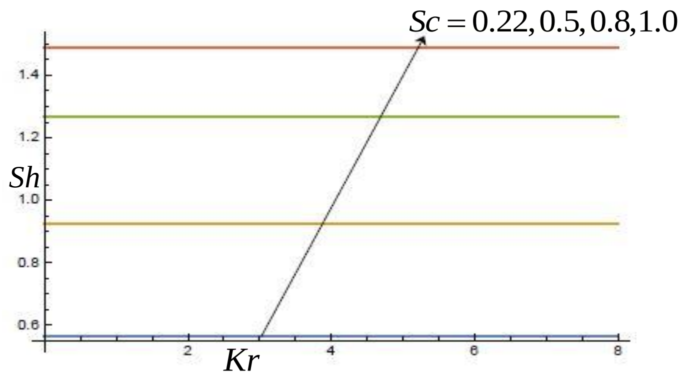

Figure 17: Influence of Schmidt number on Mass transfer rate for Sr = 0.5, ω = 0.2, ε = 0.1, n = 0.1, k_p = 1.5

5. Discussion of results

In this study, we have performed perturbation analysis on magnetohydrodynamic convective flow of a Walters'B fluid through a non-homogeneous porous medium with Soret effect. Our computed results were done with relevant physical parameters and are revealed on graphs showing the influence of the governing parameters of the flow such as Radiation Ra, Magnetic field M, chemical reaction Kr, on temperature T, velocity u, elastic parameter Rc, concentration $C,$ as well as the Nusselt number and skin friction. The various graphical results are analyzed below. Figure 1, is the effect of heat source on the velocity. It is observed that increase in the heat source decreases the velocity. The influence of thermal radiation on the velocity is depicted in figure 2. Increase in radiation increases the velocity. Physically, this shows increase dominance of conduction which increases the buoyancy force. The effect of chemical reaction on the velocity is shown in figure 3. The profile shows decrease in velocity as the chemical reaction is increased. Near the plate, the velocity peaks up and then decelerate for $y \geq 1$ to assume the free stream velocity. The influence of Soret on the velocity is shown in figure 4. The velocity increases as the Soret parameter is increased. The effect of magnetic field on velocity is presented in figure 5. The flow is retarded as the magnetic field is increased. This is due to the action of the Lorentz force which acts against the flow. Figure 6 shows the increase in the velocity as a result of increase in the permeability of the system. Physically, increase in the permeability means ability of the system to allow fluid to flow. In figure 7, velocity profiles are displayed for various values of the elastic parameter. It is observed that increase in the elastic parameter enhances the velocity.

Figure 9 shows the effect of variation of Prandtl number on the temperature. Increasing the Prandtl number reduces the temperature. The case $Pr > 1$ and in particular $Pr = 7$, corresponds to lubricating oil, polymers, for example. For these value(s), the temperature is greatly reduced. Figure 10 shows the effect of Schmidt number on the concentration of a body. An increase in Schmidt number suppresses concentration in the boundary layer. Physically, this is a manifestation of a reduction in the rate of mass diffusion. Figure 11 shows that concentration is decreased when the chemical reaction is increased. The chemical reaction decreases the concentration boundary layer. The concentration profile for variation in the Soret parameter is depicted in figure 12. For every increase in the Soret parameter, the concentration is decreased. This is a magnifiers' of increase in thermal diffusion. The influence of heat source on the skin friction is depicted in figure 13. Increase in the heat source is seen to decrease the skin friction, simultaneously; increase in radiation further increases the skin friction. In figure 14 it is observed that the magnetic field decreases the skin friction, but increase in the Grashof number increases the skin friction significantly. Figure 15 shows the effect of the elastic parameter on the skin friction. Increase in the elastic parameter increases the skin friction. Beyond $= 0.2$, the skin friction is unchanged even with increase in the frequency of oscillation. Figure 16 shows the influence of thermal radiation on the heat transfer. Increase in thermal radiation decreases heat transfer rate with further increase in heat source showing no change on the already existing effect created by the thermal radiation. The rate of mass transfer for different values of the Schmidt number is shown in figure 17. Increase in the values of the Schmidt number is accompanied by increase in the mass transfer rate. In this case, the chemical reaction offers no change in mass transfer rate even when increased to significant levels.

6. Summary

The following conclusions to this study are in order:

i. The velocity profile is increased with an increase in radiation, Schmidt number, elastic parameter, permeability and Soret, while chemical reaction decreases the velocity profile.

ii. The fluid temperature is increased as thermal radiation and heat source are improved.

iii. From the result, it is obvious that the Schmidt number affects the temperature distribution. Hence, when there is an increase in Schmidt number it causes the fluid temperature to decrease throughout the boundary layout. Just like the Grashof number, whose increase decreases the temperature distribution.

iv. More so, chemical reaction plays a vital role on the concentration profile. Thus, increase in chemical reaction reduces the concentration of species in the boundary layer.

v. Increase in radiation parameter leads to a corresponding increase in concentration. The rate of heat transfer is seen to increase by increasing radiation parameter, magnetic field parameter, but decreases for increasing values of Schmidt number.

vi. Increase in radiation leads to an increase skin friction, but the skin friction remains unchanged for increasing chemical reaction parameter.

vii. Increase in Grashof number and porosity parameter increases skin friction, while increase in radiation parameter leads to increase in skin friction.

7. Conclusion

The study has shown that thermal radiation and magnetic field have effect on velocity, temperature and concentration distribution in the flow, with significant effect on heat and mass transfer of a Walters'B flow. Also, in the absence of Soret and thermal radiation, our results compare favorably to work of Reddy *et al* (2016)

References

Amos, E. & Israel-Cookey, C. (2015). Magnetic and radiation absorption effects on thermosolutal convection on a horizontal porous layer subject to cross fluxes of heat and mass. *International journal of current research* 7(7); 18328-18339.

Crane, L.J (1970). Flow pasta a stretching plate. *Z AngewMath.Phy*, 21, 645-647.

Ezzat, M.A. & Abd-Elaal, M.Z. (1997).State space approach to viscoelastic fluid flow of hydromagnetic fluctuating boundary-layer through a porous medium. *ZAAM Journal of Applied Mathematics*, 77(5): 197-207

Ezzat, M.A. & Abd-Elaal, M.Z. (2011). Combine Heat and Mass transfer for Unsteady MHD flow of a perfect conducting micro polar fluid with thermal relaxation. *Journal of Heat and Mass Transfer,* 41(3):189-198

Kesavaiah, D.C.H., Satyanarayana, P.V. & Venkataramana, S. (2011). Effect of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction. *International Journal of Applied Mathematics and Mechanics,* 7(1), 52-69.

Magdy, A. E. & Abd-Elaal, M.Z. (1997). Free convection effects on a viscoelastic boundary layer flow with one relaxation time through a porous medium. *Journal of the Franklin Institute*. 155(1):165-177.

Makanda, G., Makinde, O.D. & Precious, S. (2013). Natural convection of viscoelastic fluid from a cone embedded in a porous medium with viscous dissipation mathematical problems in engineering. *Journal of Applied Mathematics and Mechanics*, 12(4), 89-119.

Muthucumaraswamy, R. & Ganesan, P. (2001).First-order chemical reaction on flow past an impulsively started vertical plate with uniform heat and mass flux. *ASME J. Appl. Mech.* 147(4): 45-57

Prasad, K.V. & Subhas, A.M. (2003). Diffusion of chemically reactive species of non-Newtonian fluid immersed in a porous medium over a stretching sheet. *International Journal of Non-Linear Mechanics,* 38(5), 651-657.

Raju, M.C. (2011). Radiation and mass transfer effects on a free convection flow through a porous medium bounded by a vertical surface. *Journal of Future Engineering and Technology*, 7(2), 66-75.

Ravikumar, V. and Raju, M. C. (2013).Combined effects of heat absorption and MHD on convective Rivlin – Ericksen flow past a semi – infinite vertical porous plate with variable temperature.*Ain Shams Engineering Journal* (25), 89-10

Reddy, H. S., Raju M. C. & Varma S. V. K. (2013), Chemical reaction and radiation effects on MHD free convection flow through a porous medium bounded by a vertical surface with constant heat and mass flux, *Journal of computational and Applied research in Mechanical Engineering* 3 (1); 53-62.

Reddy S. H, Raju M. C and Reddy E. K (2016), Magneto convective flow of a non-Newtonian fluid through non-homogeneous porous medium past a porous plate with variable suction. *Journal of Applied Mathematics and Physics*, 4, 233-248.

Venkateswarlu, B. & Satya, N. P (2014). Chemical reaction and radiation absorption effects on the flow and heat transfer of a nano fluid in a rotating system. *AMSE Journal of heat transfer: 128 240 - 258*

Wang, M.K.A. & Anuar, I. (1988).Unsteady viscous flow over a shrinking cylinder. *Journal of Lary Saud University-Science*. 12, 143-179.

Waqar, K., Fazle, M. and Aim, I. (2015). MHD stagnation point flow and heat transfer impinging on stretching sheet with chemical reaction and transpiration. *The chemical Engineering journal*, 24(5): 167 - 175.

Appendix

$Q = \frac{P_r}{1+R_a}$; $L = \frac{4m^2K_p+4+iwK_p}{4K_p}$, $\beta_1 = \frac{Q+\sqrt{Q^2-4Qs}}{2}$, $\beta_2 = \frac{Q+\sqrt{Q(Q-4Qs+iw)}}{2}$

$$\beta_3 = \frac{S_c + \sqrt{S_c(S_c + 4K_r)}}{2},$$

$$\beta_4 = \frac{S_c + \sqrt{S_c\left\{S_c + 4\left(K_r + \frac{iw}{4}\right)\right\}}}{2}, \quad \beta_5 = \frac{1 + \sqrt{1 + 4\left(m + \frac{1}{K_p}\right)}}{2}, \beta_6 = \frac{1 + \sqrt{1 + 4\left(m + \frac{1}{K_p}\right)}}{2}$$

$$\beta_7 = \frac{1 + \sqrt{1 + 4L}}{2}, \quad \beta_8 = \frac{1 + \sqrt{1 + 4L}}{2}, \quad D_0 = \frac{-\beta_1}{\beta_1^2 - Q\beta_1 + Q\left(S - \frac{iw}{4}\right)}, \quad D_1 = \frac{-S_cS_r\beta_1^2}{\beta_1^2 - S_c\beta_1 - S_cK_r}, \quad D_3 = \frac{\beta_3\ (1 - D_1)}{\beta_3^2 - S_c\beta_3 - \left(K_r + \frac{iw}{4}\right)S_c},$$

$$D_4 = \frac{-S_r\beta_2^2(1 - D_0)}{\beta_2^2 - S_c\beta_2 - \left(K_r + \frac{iw}{4}\right)S_c}, \quad D_5 = \frac{(D_1 - S_r\beta_1D_0)\beta_1}{\beta_1^2 - S_c\beta_1 - \left(K_r + \frac{iw}{4}\right)S_c}, \quad D_6 = \frac{-(G_r + G_cD_1)}{\beta_1^2 - \beta_1 - \left(m + \frac{1}{K_p}\right)}$$

$$D_7 = \frac{-G_c(1 - D_1)}{\beta_3^2 - \beta_3 - \left(m + \frac{1}{K_p}\right)}, D_8 = \frac{-\beta_5^2(1 - D_6 - D_7)}{\beta_5^2 - \beta_5 - \left(m + \frac{1}{K_p}\right)}, \quad D_9 = \frac{-\beta_1^2D_6}{\beta_1^2 - \beta_1 - \left(m + \frac{1}{K_p}\right)},$$

$$D_{10} = \frac{-\beta_3^2D_7}{\beta_3^2 - \beta_3 - \left(m + \frac{1}{K_p}\right)}$$

$$D_{11} = \frac{-\left(\beta_1D_6 + G_rD_0 + G_cD_5 + \frac{1}{K_p}D_6\right)}{\beta_1^2 - \beta_1 - \left(m^2 + \frac{1}{K_p} + \frac{iw}{4}\right)}, \quad D_{12} = \frac{-\{(1 - D_0)G_r + G_cD_4\}}{\beta_2^2 - \beta_2 - \left(m^2 + \frac{1}{K_p} + \frac{iw}{4}\right)},$$

$$D_{13} = \frac{-G_cD_3}{\beta_3^2 - \beta_3 - \left(m^2 + \frac{1}{K_p} + \frac{iw}{4}\right)}$$

$$D_{14} = \frac{-G_c(1 - D_3 - D_4 - D_5)}{\beta_4^2 - \beta_4 - \left(m^2 + \frac{1}{K_p} + \frac{iw}{4}\right)}, \quad D_{15} = \frac{(1 - D_6 - D_7)\left(-\beta_5 - \frac{1}{K_p}\right)}{\beta_5^2 - \beta_5 - \left(m^2 + \frac{1}{K_p} + \frac{iw}{4}\right)},$$

$$D_{16} = \frac{-\left(\beta_8D_7 + \frac{1}{K_p}D_7\right)}{\beta_8^2 - \beta_8 - \left(m^2 + \frac{1}{K_p} + \frac{iw}{4}\right)}$$

$$D_{17} = \frac{\left(\beta_1 - \frac{1}{K_p}\right) D_9 + (D_6 + D_{11})\beta_1^3 + \frac{iw}{4}\beta_1^2 D_{11}}{\beta_1^2 - \beta_1 - \left(m^2 + \frac{1}{K_p} + \frac{iw}{4}\right)}, \qquad D_{18} = \frac{\left(\frac{iw}{4}\beta_2^2 - \beta_2^3\right) D_{12}}{\beta_2^2 - \beta_2 - \left(m^2 + \frac{1}{K_p} + \frac{iw}{4}\right)},$$

$$D_{19} = \frac{\left(\beta_3 - \frac{1}{K_p}\right) D_{10} + \left(\frac{iw}{4}\beta_3^2 + \beta_3^3\right) D_{13}}{\beta_3^2 - \beta_3 - \left(m^2 + \frac{1}{K_p} + \frac{iw}{4}\right)}$$

$$D_{20} = \frac{\left(\frac{iw}{4} + \beta_4^2 + \beta_4^3\right) D_{14}}{\beta_4^2 - \beta_4 - \left(m^2 + \frac{1}{K_p} + \frac{iw}{4}\right)}, D_{21} = \frac{\left(\beta_5 - \frac{1}{K_p}\right) D_8 + (1 - D_6 - D_7)\beta_5^3 + \left(\frac{iw}{4}\beta_5^2 + \beta_5^3\right) D_{15}}{\beta_5^2 - \beta_5 - \left(m^2 + \frac{1}{K_p} + \frac{iw}{4}\right)},$$

$$D_{22} = \frac{\left(\beta_6 - \frac{1}{K_p}\right)(1 - D_8 - D_9 - D_{10})}{\beta_6^2 - \beta_6 - \left(m^2 + \frac{1}{K_p} + \frac{iw}{4}\right)}$$

$$D_{23} = \frac{\left(\frac{iw}{4}\beta_7^2 + \beta_7^3\right)(1 - D_{11} - D_{12} - D_{13} - D_{14} - D_{15} - D_{16})}{\beta_7^2 - \beta_7 - \left(m^2 + \frac{1}{K_p} + \frac{iw}{4}\right)}, D_{24} = \frac{(D_7 + D_{16})\beta_8^3 + \frac{iw}{4}\beta_8^2 D_{16}}{\beta_8^2 - \beta_8 - \left(m^2 + \frac{1}{K_p} + \frac{iw}{4}\right)}$$